Endogenous Uncertainty, CEO Compensation and Firm Performance: The Role of Firm Size*

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Abstract

We analyze the impact of firm size on the CEO contract and firm performance in the presence of endogenous uncertainty, i.e., where CEO effort affects both the expected value and variance of firm performance. Our model explains two important empirical results: (1) the negative relation between firm size and CEO compensation, and (2) the negative impact of excessive incentives on CEO performance. A key implication of our paper is that the larger the firm is, the more careful it must be in designing the CEO contract since even a small deviation above the equilibrium contract can induce reduced effort.

Keywords: principal-agent model; endogenous uncertainty; CEO compensation; firm size

JEL classification: D8, L2

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1 Introduction

Our paper focuses on how firm size affects the CEO contract and effort in the presence of endogenous uncertainty. In the standard principal-agent model, the agent who acts as a CEO is exposed to exogenous uncertainty. That is, her effort increases the expected firm output but is not linked to the variance of that output. In contrast, endogenous uncertainty implies that increasing CEO effort increases not only the expected firm output but also the riskiness that the firm is exposed to. This creates a dilemma to a risk-averse CEO since her effort now affects the mean-variance tradeoff. In the model with exogenous uncertainty, this dilemma does not arise.

In our paper, first, we show that, in the model with endogenous uncertainty, the equilibrium CEO incentives decline as firm size increases. This relationship has been confirmed by several empirical studies, but is not captured by the model with exogenous uncertainty.

Second, we show that in the model with endogenous uncertainty, excessive (that is, non-equilbrium) incentives for large firms can lead to decreased CEO effort, and correspondingly, poorer firm performance. This is consistent with the empirically documented inverted-U relationship between incentives and firm performance, a phenomenon that the model with exogenous uncertainty is unable to replicate. Introducing endogenous uncertainty generates negative feedback to CEO effort such that excessive incentives may induce the CEO to reduce effort. Including the firm size as a variable in this model allows us to look at whether positive deviations from the equilibrium would be more likely to negatively affect CEO effort in large or small firms. We find that excessive incentives are more likely to lead to declining CEO effort in large firms, and that as firms get very large even tiny deviations above the equilibrium contract can lead to decreased effort.

A key implication of our paper is that large firms must be more careful than small firms when it comes to contract design. This is because small deviations above the equilibrium contract are more likely to lead to declining CEO effort in large firms than in small firms.

2 Modeling Firm Size and CEO Compensation

We use the modeling framework in Schaefer (1998) for looking at the relationship between firm size and optimal incentive level. This framework characterizes the typical agency model, and we can easily incorporate endogeneity into the model. Suppose that the shareholder is the principal (he) and the CEO is the agent (she). Schaefer (1998) assumes that CEO effort

generates benefit, B, defined as

$$B = Sa + S\varepsilon, \tag{1}$$

where S is firm size, a is unobserved CEO effort, and ε is a normally distributed random variable with mean zero and variance σ^2. We assume that the contract is linear[1] and consists of the CEO's fixed wage, α, and share of profits, β. We assume that the principal is risk-neutral. The CEO's cost for effort is characterized as a quadratic cost function, $\frac{1}{2}ca^2$. She is risk-averse, and represented by a negative exponential utility function with a coefficient of risk aversion, η. We will call this the model with exogenous uncertainty since CEO behavior does not affect the variance of output.

The CEO determines effort by optimizing her objective function:

$$V_{CEO} = \alpha + \beta Sa - \frac{1}{2}\eta\sigma^2\beta^2S^2 - \frac{1}{2}ca^2, \tag{2}$$

in which, as we will see when we discuss the model with endogenous uncertainty, the most interesting element is $\frac{1}{2}\eta\sigma^2\beta^2S^2$. This element captures the fact that the agent is exposed to risk which she dislikes and we refer to it as the "cost of exposure to risk."

The agent's best-response function is linear:

$$\hat{a} = \frac{\beta S}{c}. \tag{3}$$

Given the participation constraint for meeting the CEO's reservation wage, R, the shareholder's value function becomes:

$$V_{SH} = -R + Sa - \frac{1}{2}\eta\sigma^2\beta^2S^2 - \frac{1}{2}ca^2. \tag{4}$$

As given in Schaefer (1998), the equilibrium values of the agent's share and effort are:

$$\hat{\beta}^* = \frac{1}{1 + c\eta\sigma^2}, \tag{5}$$

[1]While linear contracts may not be optimal (see, for instance, Mirrlees (1974) and Mirrlees (1999)), they are very popular in real life (see, for instance, Holmstrom and Milgrom (1987), Schmalensee (1989), Milgrom and Roberts (1992), and Bhattacharyya and Lafontaine (1995)) and the literature. Our key objective is to analyze whether or not introducing endogenous uncertainty allows to explain real life phenomena which the model with exogenous uncertainty is unable to capture. In addition, we build our model on and compare to Schaefer (1998) who relies on the assumption that contracts are linear. Consequently, it is only natural that we focus our attention on optimal linear contracts.

$$\hat{a}^* = \frac{S}{c(1+c\eta\sigma^2)}. \tag{6}$$

Now we introduce endogenous uncertainty into the above model. First, note that in the model with exogenous uncertainty the expected value and variance of outcome are $E(B) = Sa$ and $V(B) = S^2\sigma^2$, respectively. Given that $E(B) = Sa$, there are two ways to increase the expected value of outcome: either increase the firm's size or increase agent's effort. If S increases, then $V(B)$ increases as does the expected value of output, and we obtain the mean-variance tradeoff. However, if the agent increases her effort, then nothing happens to the variance of B. We endogenize the variance by defining B in the following way:

$$B = Sa + Sa\varepsilon, \tag{7}$$

Note that while the expected value of B is still $E(B) = Sa$, the variance becomes $V(B) = S^2a^2\sigma^2$. Consequently, in our model, no matter what causes $E(B)$ to increase, firm size or effort, we also obtain an increase in $V(B)$.

The motivation for assuming a positive relationship between CEO effort and variance is twofold. First, this case is of particular interest to us since the CEO explicitly faces a trade-off between an increase in expected output and an increase in riskiness of output. Tying the riskiness to the effort level implies that the CEO is essentially selecting the riskiness of the project. As identified in prior research (e.g., Lambert (1986) and Garen (1994)), a potential for conflict of interest between the CEO and shareholder arises when the CEO can select the riskiness of the project since the CEO may want to limit risk exposure by selecting a project with lower variance even when that project has a lower expected outcome than another project. Second, this assumption is similar to that of Schaefer (1998) in that he argues that the variance should be increasing with the expected value of output. However, he considers the variance to be proportional to the fixed output of the previous year, whereas our model is static so that output is simply proportional to total output, which depends on effort.

In our model, the CEO objective function becomes:

$$V_{CEO} = \alpha + \beta Sa - \frac{1}{2}\eta\sigma^2\beta^2S^2a^2 - \frac{1}{2}ca^2, \tag{8}$$

where the main difference from (2) is that the agent takes into account the negative impact of her effort on the variance of the outcome. This is captured by the cost of exposure to risk which, in the model with endogenous uncertainty, includes a^2.

The CEO best-response function, $\tilde{a}$, is no longer a linear function of β,

$$\tilde{a} = \frac{\beta S}{c + \eta\sigma^2\beta^2 S^2}, \tag{9}$$

and we analyze it in detail in subsection 3.2. The shareholder's objective function becomes:

$$V_{SH} = -R + Sa - \frac{1}{2}\eta\sigma^2\beta^2 S^2 a^2 - \frac{1}{2}ca^2. \tag{10}$$

We obtain the equilibrium CEO share and effort.

Proposition 2.1

$$\tilde{\beta}^* = \frac{\sqrt{c^2 + 4c\eta\sigma^2 S^2} - c}{2\eta\sigma^2 S^2} \tag{11}$$

$$\tilde{a}^* = \frac{S}{\sqrt{c^2 + 4c\eta\sigma^2 S^2}} \tag{12}$$

3 Exogenous and Endogenous Uncertainty: Model Implications

3.1 Firm Size and Equilibrium Incentives

Empirical studies have found that CEO incentives for improving corporate performance decrease as firm size increases (e.g., Jensen and Murphy (1990) and Schaefer (1998)). There have been various theoretical attempts to explain this compensation behavior. Haubrich (1994) provides calibrations showing that Jensen and Murphy's findings are consistent with optimal contracting if CEOs are sufficiently risk-averse. Edmans et al. (2009) use a multiplicative effect model to argue that the correct measure of returns on effort is the impact of CEO effort on the percent growth of the firm, and in turn, the CEO earns a percent increase in remuneration. Dicks (2012) argues that larger firms have more effective governance, such that monitoring substitutes the need for monetary incentives as firms get larger.

We now test how firm size affects the equilibrium share of the agent. Figure 1 illustrates, setting all other parameter values to one, how equilibrium share changes with respect to firm size under the exogenous (dashed line) and endogenous (solid line) uncertainty, respectively.

[Figure 1 about here.]

In the model with exogenous uncertainty, the CEO share is constant with respect to firm size: as we see from (5), the derivative of $\hat{\beta}^*$ with respect to S is zero. That is, under these assumptions the CEO is given the same incentive level no matter how big the firm is. However, when uncertainty is endogenous, then the relationship between firm size and CEO incentives becomes negative. We note that

$$\frac{d\tilde{\beta}^*}{dS} = \frac{c\left[\sqrt{c^2 + 4c\eta\sigma^2S^2} - (c + 2\eta\sigma^2S^2)\right]}{\eta\sigma^2S^3\sqrt{c^2 + 4c\eta\sigma^2S^2}} \tag{13}$$

and it is easy to verify that $\sqrt{c^2 + 4c\eta\sigma^2S^2} < c + 2\eta\sigma^2S^2$. Thus our model predicts that CEOs of larger firms will have lower incentives, consistent with what has been found in the empirical literature.

To understand why this prediction is made by the model with endogenous uncertainty but not by the model with exogenous uncertainty, we need to look at the CEO objective functions in both models (equations (2) and (8)). In each model, an increase in S has a positive and a negative impact on V_{CEO}: βSa increases (which the agent likes) but, simultaneously, so does the cost of exposure to risk (which the agent dislikes). In addition, an increase in S implies that the equilibrium value of agent effort increases. Note that in the model with exogenous uncertainty, the cost of exposure to risk is unaffected by changes in effort. In each model, it is necessary to balance the positive and negative consequences of increasing firm size. This balance is achieved by manipulating the agent's share. In the model with exogenous uncertainty, we can informally say that the negative impact of increasing S is smaller since the agent's effort (which increases due to an increase of S) does not affect the cost of exposure to risk. Consequently, in this model it is enough that when S increases, the agent's share remains the same. However, in the model with endogenous uncertainty, the negative impact of increasing S is greater since the agent's effort increases the cost of exposure to risk. Hence, compared with the model with exogenous uncertainty, a more drastic response is required, and the agent's share decreases.

Jensen and Murphy (1990) argue that the incentives for CEOs found in their empirical estimates are excessively low and should be increased in order to better align the interests of the CEO with that of shareholders. To make their point, they give the example that an important aspect of franchises is to provide incentives to the local franchiser to maximize effort through 100% ownership. Extending that intuition to the CEO, they argue that the CEO would provide the highest effort if she receives 100% of the benefits of her effort. Figure 1 illustrates that Jensen and Murphy are likely to be correct in their assertion that franchisees

provide a close to optimal level of effort when they have full personal ownership, since owning and managing one or several outlets may reflect a relatively "small" firm. However, the same logic does not extend to large firms. The firms Jensen and Murphy were studying in their paper were very large, so our analysis suggests that CEO incentives should be small for these firms. However to better address their recommendation, we need to consider what might happen if firms were to increase CEO incentives. That is the subject of the next subsection.

3.2 The Relationship between Incentives and Performance

A second empirical finding relevant to this paper is an observed inverted-U relationship between CEO ownership and firm performance (Morck et al. (1988), McConnell and Servaes (1990), Hermalin and Weisbach (1991), Hubbard and Palia (1995), Holderness et al. (1999), Fogelberg and Griffith (2000), Claessens et al. (2002), Anderson and Reeb (2003), Tian (2004), Davies et al. (2005), Adams and Santos (2006), Pukthuanthong et al. (2007), McConnell et al. (2008), Benson and Davidson (2009), Fahlenbrach and Stulz (2009), Kim and Lu (2011), and Zhang (2013)). The positive relationship between CEO incentives and firm performance is presumed to arise because of convergence of interest between the principal and agent as the agent owns more of the firm (e.g., Morck et al. (1988)). Convergence of interest implies that as their ownership of the firm increases, CEOs have greater financial incentives to increase effort to ensure that firm profits are optimized. The negative relationship between ownership and firm performance is typically explained by the risk-reduction hypothesis (e.g., Benson and Davidson (2009)) according to which if the firm increases the manager's incentives, then her exposure to risk increases and she may respond by reducing effort since this action lowers her risk exposure.

We now examine how firm performance is related to the CEO's incentives in the models with exogenous and endogenous uncertainty. In the former, expressing CEO effort in terms of the share she receives yields the best response function given in equation (3) which is a linear function of the agent's share. However, when uncertainty is endogenous, increasing incentives may not always increase effort. Expressing effort as a function of the share that the CEO receives yields equation (9). Taking the derivative of $\tilde{a}$ with respect to the agent's share reveals that $\tilde{a}$ is increasing in β whenever $\beta < \bar{\beta} = \sqrt{\frac{c}{\eta\sigma^2 S^2}}$ and decreasing in β whenever $\beta > \bar{\beta}$. In order for CEO effort to be decreasing with incentives, it is necessary that $\bar{\beta} < 1$ which implies that we need $S > \sqrt{\frac{c}{\eta\sigma^2}}$. In other words, effort will decline with incentives only if the firm is large enough. The decreasing effort with respect to the CEO's share arises because the agent's effort and firm size interact to generate excessive risk at high levels of

agent's share. The agent responds by reducing effort, which in turn reduces risk. Our line of reasoning is consistent with the risk-reduction hypothesis proposed in the literature.

Given the assertion by Jensen and Murphy (1990) that CEO incentives should be increased in order to better align their effort with shareholder needs, we should ask to what extent shareholders need to worry about excessive incentives leading to declining effort. In particular, this means that we need to analyze the difference between the threshold value of the agent's incentives, $\bar{\beta}$, and the equilibrium value of the agent's incentives, $\tilde{\beta}^*$. There are two important characteristics of the difference $\bar{\beta} - \tilde{\beta}^*$. First, the difference is decreasing as firm size increases. This means that given a fixed increase in the agent's incentives, larger firms are more likely to offer excessive incentives, that is, incentives above $\bar{\beta}$ that induce the CEO to reduce effort. Moreover, it can be shown that the difference $\bar{\beta} - \tilde{\beta}^*$ converges to zero as S goes to infinity. Consequently, even if we consider a very small positive deviation from equilibrium, this can lead to counter-productive CEO behavior if the firm is large enough. Proposition 3.1 summarizes these two points.

Proposition 3.1

$$\frac{\partial\left(\bar{\beta} - \tilde{\beta}^*\right)}{\partial S} < 0 \tag{14}$$

$$\lim_{S\to\infty} \bar{\beta} - \tilde{\beta}^* = 0 \tag{15}$$

The above suggests that while deviations in incentives above the equilibrium level may not have negative impacts on CEO effort for small firms, even tiny deviations above the equilibrium contract may lead to decreased effort for very large firms. Thus the larger the firm is, the more important it becomes to correctly identify the equilibrium contract in order to avoid inducing declining effort on the part of the CEO. In other words, contract design is a more delicate operation in large firms than small firms.

4 Conclusions

In this paper we combined firm size with endogenous uncertainty in the agency model. We find that the optimal incentives given to CEOs decline with firm size, with equilibrium incentives being quite small for very large firms, consistent with the findings in the empirical literature. Moreover, for large firms, increasing the incentives even slightly above the equilibrium levels may lead to counter-productive impacts on effort, misaligning the interests of

the agent from that of the shareholders. Thus counter-productive excessive incentives are a more serious concern for large firms.

Given risk-aversion, it is natural that agents will consider the feedback effects of their own effort on the variance of outcomes. Our results illustrate the importance of considering endogeneity in the uncertainty faced by CEOs, and provide another caution against assuming a positive relationship between incentives and effort.

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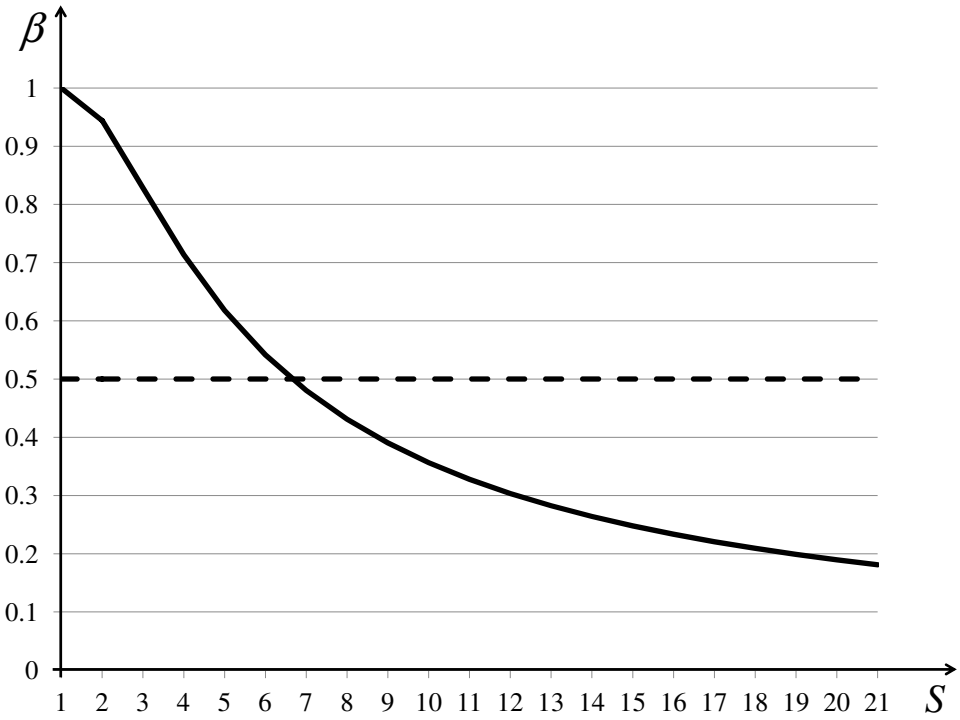
β
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
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Figure 1