

More is Not Always Better: the Case of Counter-Terrorism Security*

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Abstract

Can counter-terrorism security be counter-productive? We argue that it can be when the at-risk population acts strategically. We model a two-stage game where the government first chooses the defensive security level for a public place. The second stage is a simultaneous-move game with terrorist choosing terror effort and members of the population deciding whether or not to attend the public place. Our key measure of the efficiency of the counter-terrorism security is the expected number of casualties. Under very standard and general assumptions, we show that it is possible that more security leads to an increase in that number. This is because increasing security both discourages and encourages the terrorist. On the one hand, more security makes a successful terror attack less likely (discouragement). On the other hand, more security motivates more people to attend the public place which makes the attack more valuable to the terrorist (encouragement).

Keywords: terrorism, counter-terrorism security

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1 Introduction

According to the Oxford English Dictionary, terrorism is “the unlawful use of violence and intimidation, especially against civilians, in the pursuit of political aims.”¹ Because of its violent nature and targeting civilian population, the fear of terrorism is among the top worries of the modern world. In June 2017, 42% of Americans were worried about being a victim of terrorism and 66% of them consider the possibility of future terror attacks in the U.S. as a great or fair concern.² In 2016, 79% of American respondents pointed at international terrorism as a critical threat to the United States making it number one concern in the nation (development on nuclear weapons by Iran was the second highest with 75%).³

Given that the fear of terrorism is widespread and significantly affects people’s behavior, it is of utmost importance to incorporate it into the studies of terrorism. The impact on behavior can be seen in a June 2017 poll which shows that “as a result of the events relating to terrorism in recent years” 32% American indicated that they are less willing to fly airplanes, 26% were less willing to go to skyscrapers, 38% less willing to attend an event with thousands of people, and 46% less willing to travel abroad.⁴

Activities like going to a public place (e.g., 2008 Mumbai attacks), attending a public event (e.g., 2017 Ariana Grande concert in Manchester), or using public transportation (e.g., 2004 Madrid train bombing) are associated with the risk of being targeted by terrorists. Whatever is “public” is a potential target of a terror attack whose objective is to inflict a maximal loss of life. When deciding whether or not to be part of a “public” event and go where expected crowd is to be, people take the risk of terror attack into account and might opt out.

Our focus on “public places” is motivated by Brandt and Sandler (2011) who show that terrorists are now concentrating on attacking public places. There is also empirical evidence which supports

¹Sandler (2015) provides a detailed discussion on the definitions of terrorism used in the literature.

²Source: Gallup poll, <http://news.gallup.com/poll/4909/terrorism-united-states.aspx>. Retrieved on January 12, 2018.

³Source: Gallup poll, <http://news.gallup.com/poll/189161/americans-cite-cyberterrorism-among-top-three-threats.aspx>. Retrieved on January 12, 2018.

⁴Source: Gallup poll, <http://news.gallup.com/poll/4909/terrorism-united-states.aspx>. Retrieved on January 12, 2018.

the fact that people change behavior as a response to terror. Enders and Sandler (1991), Enders et al. (1992) and Drakos and Kutan (2003) find evidence of tourists not favoring destinations with higher risk of terrorism. Elias et al. (2013) show that commuters in Israel choose private means of transport over public transport as their risk perception of terror activities increase. Kalist (2010) documents that in the US, as the terror-alert levels initially went up after the September 2001 attacks, there was a dip in attendance for Major League Baseball events. Abadie and Gardeazabal (2008) show how investors strategically move away from countries and regions with higher terrorist risk. Sandler and Enders (2008) provide an excellent review of economic consequences of terrorism. In order to address the fear of terrorism and prevent terror attacks, governments around the world spend large amounts of taxpayer money on defensive counter-terrorism security.⁵ For example, in USA, that amount is around \$100 billion per year (Mueller and Stewart (2014)). It is only natural to ask about the effectiveness of counter-terrorism security.

Suppose that the counter-terrorism security is costless. Is more security always better?

Instinctively, the answer is “yes.” After all, more security makes a public place⁶ safer. However, as we explore in this paper, the problem is more complex than what instinct would indicate and it is not unlikely that the answer is “no.”

We analyze the counter-terrorism security in the context of saving lives. In order to address our question, we model a strategic interaction between the government, terrorist, and population whose members the government wants to protect against the terror attack. The government acts first and provides security that works to directly reduce the effectiveness of the terror attack in the subsequent period. In the next period, both terrorist and population members act simultaneously. Many game theoretic models of terrorism consider the case where the government acts first and chooses its security level which negatively impacts the effectiveness of terrorist activity followed by

⁵We focus on defensive counter-terrorism security (e.g., Bandyopadhyay and Sandler (2011), Brandt and Sandler (2011)) rather than proactive counter-terrorism security. Hence, whenever we use the term “security,” it should be understood as “defensive security.”

⁶In our paper, the term “public place” is not restricted only to physical places but also includes any event or activity that is open to the public and potentially attracts the crowds (e.g., concert, public transportation, market place, tourist attraction). It is also plausible to think of “public place” as the whole public sphere – all public infrastructure and events; in this context, counter-terrorism security should be understood at the national level.

the terrorists choosing their action. Papers where the government acts first and allocates resources towards security include Bueno De Mesquita (2005), Bueno De Mesquita (2007), Bandyopadhyay et al. (2011), and Carter (2015). Papers looking at the optimal security provision include notable work by Rosendorff and Sandler (2004), Siqueira and Sandler (2006), Sandler and Siqueira (2006), Farrow (2007), Bueno De Mesquita (2007), and Bandyopadhyay and Sandler (2011). Note that these papers do not account for a strategic population and the impact of government choice on both terrorist and members of the population.

In line with mainstream research, we assume that terrorists are rational.⁷ They choose the level of terror effort which directly increases the probability of terror attack being successful. The effort level is chosen to maximize the terrorists' objective function which is the expected impact of terror (measured by the number of casualties) less the cost of terror effort. The attack targets a public place. The more populated the public place is, the more attractive it is for the terrorist. Hence, the terrorist takes into account how many people plan to be at the public place and the security level chosen by the government. There has been a lot of game theoretic work on terrorism in general where the assumption of the rational terrorist is standard. Excellent reviews of this literature can be found in Sandler and Arce (2003), Bueno De Mesquita (2008), and Sandler and Siqueira (2009).

We consider a strategic population where each person chooses whether or not to expose themselves to terror risk. For instance, people choose to stay at home or go to a concert, live in the suburbs or the city, and travel by car or use public transport. In all of these cases, people choose between a relatively safer option with very low terrorism threat and one with high terror threat. By attending the public place, each person takes the risk of not getting any value if the terror attack is successful or getting a high value in case it is not. By not attending the public place, population members get their respective reservation values. The choice of the population members depends on the government's level of counter-terrorism measures and terrorist's effort.

Note that the assumption of the strategic population is where this paper departs from the mainstream game theoretic literature on terrorism. There has been some previous work where a strategic population was assumed. Keohane and Zeckhauser (2003) consider a game theoretic model with a

⁷Some authors, for example, Abrahms (2008), posit that terrorist need not be rational agents.

strategic population which chooses to remove itself from the site of intended terror attack. Given a fixed disutility of harm from terror attack and a cost of switching to a safe haven, they show that if probability of a successful attack is increasing in the number of people at the location then the number of people at the site of the terror attack will be such that the harm from terror is just equal to the cost of the other location. In this homogenous setting, government measures to reduce the probability of harm or the disutility from harm are both ineffective to change the total welfare. Our model offers very different insight by including government, terrorist and population as strategic players in a multi-stage game and by further allowing for heterogeneity in the population. An additional point of departure of this paper is in considering the government which tries to maximize the sum of utilities for entire population instead of just trying to minimize the impact of the terrorist. Another paper looking at a strategic population is Lakdawalla and Zanjani (2005) where residents choose between safe and risky options. They look at the case where the choice of safe options by individuals creates a public bad which can be avoided by the government giving incentives for insurance.

In the simultaneous game (with fixed security level) between terrorist and population, we find that the mass of the population that exposes itself to terror risk is always increasing in the security level while the probability of a successful attack is always decreasing in the security level. However, when it comes to the terror effort, the results are more complex as the terror effort is either decreasing (expected result) or increasing (unexpected result) in the security level. This is because, counter-terrorism security acts as both deterrent and encouragement for terrorist. On the one hand, more security makes it less likely that the attack will be successful. On the other hand, more security motivates more people to attend the public place; this increase in attendance makes that very same public place a more attractive target for a terror attack.

Next, we turn our attention to the main problem of this paper: effectiveness of the government-provided security in the context of saving lives. Terrorist attacks have many negative consequences: destruction of infrastructure, decrease in foreign direct investment, disruption of financial markets, etc. While infrastructure can be rebuilt, foreign investors can return, and financial markets can bounce back, there is one target of terror attacks that can not be recovered: human life. For that

reason terrorism is currently such an important issue and that is why we focus on lost lives.

The relevant measure to consider is the expected number of casualties due to terror attack. Now, we can more precisely posit our main question:

Suppose that the counter-terrorism security is costless. If the goal is to decrease the expected number of casualties, is more security always better?

Under standard and general assumptions, we find that the relationship between the security level and expected number of casualties can be summarized in the three cases that we depict in Figure 1. Case 1 is what we desire and instinctively predict: the expected number of casualties always decreases in security. In the second case, we have the very opposite since the expected number of casualties always increases. It is Case 3 – where the expected number of casualties increases if security is too small and decreases when security is large enough (inverted U-shaped relationship) – that we find to be the most interesting and important.

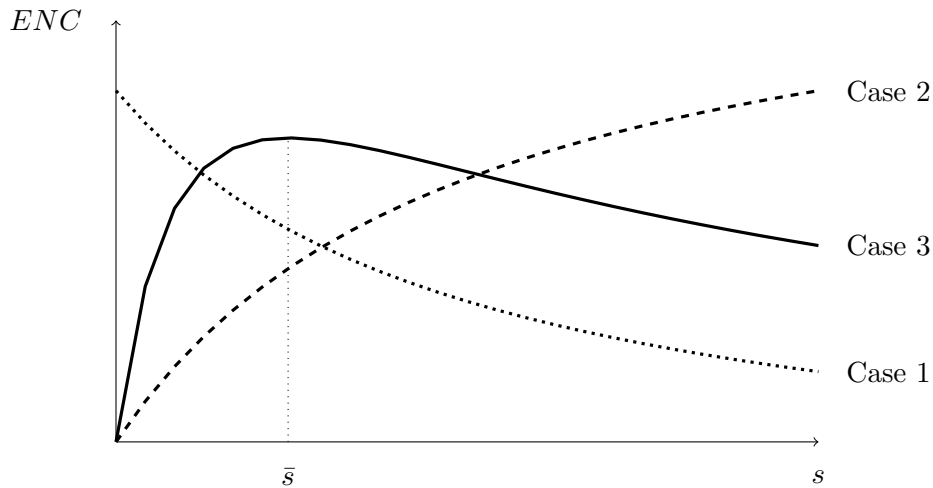


Figure 1: Counter-terrorism security (s) and expected number of casualties (ENC).

Case 3 supports the view that it is better to over-spend on counter-terrorism measures rather than to act too prudently. When the government spends too little (i.e., less than a threshold value \bar{s}), then increasing security will actually bring devastating results. However, there is no such risk with

increasing the expected number of casualties by expanding the counter-terrorism budget when the expenditures are already above the threshold level. Case 3 is also important for the government that operates under severe constraints; that is, when the maximum feasible amount of security is at most \bar{s} . With such a constraint, Case 3 is the same as Case 2 where the optimal quantity of security is zero.

In short, the answer to our question – whether more counter-terrorism security is always better – is that more is not always better and, actually, more can make things worse.

The question of optimal security provision is also dealt with in the strand of literature which considers the case of a defender protecting multiple locations against an attacker. Powell (2007a) and Powell (2007b) both consider the case of a defender choosing how to divide a fixed security resource across sites and attacker choosing how to divide the probability of attacking each of those sites. Bier et al. (2007) also consider optimal security provision across two sites. They model the probability of a successful attack as a function of government security for that site and allow the terrorist to choose which site to attack. Zhuang and Bier (2007) allow the probability of successful attack across multiple sites to vary with security level and terrorist effort as in this paper. Wang and Bier (2011) model a situation with multiple targets with incomplete information where the defender is unaware of attacker's private preferences over sites. An important result of the literature till now is that government allocates security such that the marginal loss from the two sites is equated. We would like to highlight that in our paper, we do not explicitly model two (or more) locations as we are not interested in the problem of competition in security. This is because we think of there being just one public place and our focus is on the problem of effectiveness of security in the context of strategic population. Rather than asking how much security to provide, we want to learn if more security is always beneficial. In our paper, members of the population choose between a safe and risky location. The terrorist and government effectively choose their actions on one location only. We can think of their actions being constrained for the safe location.

The next section describes the model. Section 3 looks at the second-stage game between the terrorist and the population. Section 4 analyzes the effectiveness of counter-terrorism in terms of

saving lives and welfare. Section 5 concludes. All proofs are collected in the Appendix.

2 Model

The three players of the game are the government, terrorist and population. The game consists of two rounds such that:

- In the first round government chooses counter-terrorism security level.
- The second round consists of a simultaneous game involving the terrorist and population members. Members of the population choose whether to attend the public place or not while the terrorist chooses the effort with which to attack that public place.

The game begins with the government choosing the level of security denoted by s . Next, the terrorist and members of the population act. We let t denote the level of effort chosen by the terrorist. Members of the population individually decide whether or not to attend the public place. We let m denote the total mass of population that chooses to attend the public place.

A fundamental variable in our model is the probability of a successful attack denoted by $\lambda(s, t)$. We assume that the terrorist and government both have control over this variable. The government can decrease the probability of a successful attack by increasing its security effort while the terrorist can increase that probability by increasing their terror effort. More formally, we make the following assumptions about the $\lambda(s, t)$. Note, however, that we make no assumptions about the functional form of $\lambda(s, t)$.

Assumption 1.

$\lambda(s, t)$ is continuous in both s and t and twice continuously differentiable such that $\lambda_t > 0$, $\lambda_s < 0$, $\lambda_{tt} \leq 0$, and $\lambda_{ts} < 0$.⁸

⁸The notation used in this paper conforms to standard notation where given a function $f(x, y)$ of variables x and y , we write $f_x = \frac{\partial f}{\partial x}$, $f_{xx} = \frac{\partial^2 f}{\partial x^2}$, and $f_{xy} = \frac{\partial^2 f}{\partial x \partial y}$.

The assumptions on the probability of the successful attack ensure that the probability is increasing at a decreasing rate in the terrorist effort but decreasing in the government-provided security. Additionally, the marginal gain from terrorist effort, is decreasing in s . Hence government security dampens the overall probability of a successful attack and also the marginal impact from terrorist effort.

The terrorist seeks to maximize the expected impact of terror less the cost of terror. The members of population choose whether the utility of attending the place given the probability of a successful attack is more than that of staying away. We next discuss the terrorist's and the population's problems in detail.

Terrorist. We model a rational terrorist who maximizes a utility function which in the standard form: benefit from the terror act less the cost of execution of that act. We assume that both benefit and cost are measured in psychological units. A plausible assumption regarding the benefit from the terror act is that the terrorist's utility is rising in the expected impact of the act. We simplify this argument and assume that the terrorist benefit is exactly the expected impact of the act. We do not assume any particular functional form of the cost of the terror act except for what is typical for the literature: cost is increasing and convex in effort.

Assumption 2.

$c(t)$ is continuous and twice continuously differentiable in t such that $c' > 0$ and $c'' > 0$.

The expected impact of terror depends on the probability of a successful attack and the size of the population that attends the public place. Let m denote the mass of population attending the public place and $c(t)$ denote the cost of effort. The terrorist maximizes expected utility U_T .

$$U_T(s, t, m) = m \cdot \lambda(s, t) - c(t) \tag{1}$$

The terrorist gets utility zero if the attack is unsuccessful while if the attack is successful the terrorist's utility is exactly m which is the mass of population that was attacked. This utility captures the fact that the terrorist not only wants to launch a successful attack but also inflict

maximum possible damage. We assume that the terrorist's reservation utility is zero. This implies that the terror attacks whenever $U_T \geq 0$.

The following standard technical assumption regarding the terrorist's utility function ensures that the terrorist's optimization program has an interior solution.

Assumption 3.

$$\lim_{t \rightarrow 0} [m\lambda_t - c'] > 0 \text{ and } \lim_{t \rightarrow \infty} [m\lambda_t - c'] < 0.$$

Population. We model a strategic population of total mass one. Each member of the population has two choices — of either attending the public place and exposing themselves to terror risk or of staying away from the public place. Each member evaluates the relative utility from going to the public place versus staying away and chooses the option that gives them the higher utility.

To evaluate the utility from going to the public place, each member must consider two cases. One where the terror attack is successful and the other where it is not successful. We assume that everyone gets utility one from attending the public place in case the attack fails while if a person attends a public place and the attack is successful, then the utility is zero. Since the members of the population are maximizing expected utility, they evaluate the expected benefit from going to the public place as $1 \cdot (1 - \lambda(s, t)) + 0 \cdot (\lambda(s, t))$. Each member thus has a rational expectation of the probability of a successful attack $\lambda(s, t)$.⁹

Instead of attending the public place, an individual can stay at home. Here, we assume that the population is heterogenous in terms of utility derived from staying at home. That is, the value of staying at home is not the same for everyone. Let $\alpha \in [0, 1]$ denote the utility from staying at home.¹⁰ We make no specific assumption about the variable α other than it is distributed over $[0, 1]$ with cumulative distribution function F . Let F_α denote the first-order derivative of F , $F_\alpha := \frac{dF}{d\alpha}$.

We assume that F is continuous.

⁹We can think of members of the population forming this expectation via information from news media.

¹⁰We can think of these utility values — zero, α , and one — as being relative values. That is, we normalize the utility function in such a way that its range is between zero and one. Each individual faces three scenarios: go to the public place and not face a terror attack (highest utility value of 1), go to the public place and face a terror attack (lowest utility value of 0), or stay away from the public place (intermediate utility $\alpha \in [0, 1]$). Each member of the population chooses to go to the public place only if the value from staying away is low enough given the probability of a successful terror attack.

Assumption 4.

F is continuous.

Government. We analyze the impact of increasing counter-terrorism security on the **Expected Number of Casualties**, $m \cdot \lambda$. We also look at how security alters social welfare, U_{TW} , where social welfare is the sum of two elements: the aggregate utility of those who attend the public place, $U_1(s, t, m) = m(1 - \lambda(s, t))$, and the aggregate utility of those who stay at home, $U_2(s, t, m) = \int_{\bar{\alpha}}^1 \alpha dF$.

We solve the game by backward induction. In the next section, we solve for the second-stage of the game where government security level is fixed and the members of population and terrorist choose their strategies simultaneously. We characterize the Nash equilibrium for the second stage as a function of the level of government security. In Section 4, we then consider the problem of government choosing the security level.

3 Population and Terrorist

We begin by analysing the behaviour of the terrorist and the population given a fixed level of security s . That is, we find the equilibrium in the second-stage simultaneous-move game between population and terrorist. In this section, the government-provided security level is presumed to be fixed.

Every person chooses between staying at home and attending the public place. Staying at home generates certain utility α while attending the public place generates a lottery that yields utility zero with probability λ and utility one with probability $1 - \lambda$. Hence, the expected utility from attending the public place is $1 - \lambda$.¹¹

Given s and t , let $\bar{\alpha}(s, t)$ be the highest utility from staying at home such that everyone with utility from home less than $\bar{\alpha}$ attends the public place. Since every person maximizes expected utility,

¹¹We assume that if a person is indifferent between staying at home (utility α) and attending the public place (expected utility $1 - \lambda$), then that person attends the public place.

$\bar{\alpha}(s, t)$ is defined in the following way.

$$\bar{\alpha}(s, t) = 1 - \lambda(s, t) \quad (2)$$

Let $\hat{m}(s, t)$ denote the mass of the population that attends the public place given that the terrorist's effort is t .

$$\hat{m}(s, t) = F(\bar{\alpha}(s, t)) = F(1 - \lambda(s, t)) \quad (3)$$

Next, let $\hat{t}(s, m)$ denote terrorist's best-response function given the mass of the population attending the public place, m . There exists a unique interior solution of the terrorist's optimization program (Equation (1)). This solution, $\hat{t}(s, m)$, satisfies the following first-order condition.

$$\frac{\partial U_T}{\partial t} = m\lambda_t - c' = 0 \quad (4)$$

We turn attention to the second-stage equilibrium values, $\tilde{m}(s)$ and $\tilde{t}(s)$. Both $\tilde{m}(s)$ and $\tilde{t}(s)$ are functions of counter-terrorism security s since that security is chosen at the first stage (hence, s is fixed at the second stage). We derive $\tilde{m}(s)$ and $\tilde{t}(s)$ from the following system of two equations.

$$\tilde{m}(s) = \hat{m}(s, \tilde{t}(s)) \quad (5)$$

$$\tilde{t}(s) = \hat{t}(s, \tilde{m}(s)) \quad (6)$$

Our first proposition analyses the behavior of second-stage equilibrium values: population attending the public place $\tilde{m}(s)$, terrorist effort $\tilde{t}(s)$, and the probability of a successful attack $\tilde{\lambda}(s)$.

Proposition A.

The second-stage equilibrium is such that:

1. $\frac{d\tilde{m}(s)}{ds} \geq 0$ for each s .

2. $\frac{d\hat{t}(s)}{ds} \leq (\geq) 0$ for each s , if $F(1 - \lambda) \cdot \lambda_{ts} - F_\alpha(1 - \lambda) \cdot \lambda_s \cdot \lambda_t \geq (\leq) 0$ for each pair (s, t) .
3. $\frac{d\tilde{\lambda}(s)}{ds} < 0$ for each s .

The intuition for Proposition A1 lies in two facts based on the best-response functions, $\hat{m}(s, t)$ and $\hat{t}(s, m)$. First, keeping terrorist activity constant, the mass of population attending the public place is increasing with security (i.e., $\frac{\partial \hat{m}}{\partial s} \geq 0$, see Lemma 1 in the Appendix). Second, increased security leads to the terrorist reducing effort for each fixed size of population attending the public place (i.e., $\frac{\partial \hat{t}}{\partial s} < 0$, see Lemma 2 in the Appendix) and this reduction in terrorist effort leads to a greater mass of population attending the public place.

Proposition A2 shows that the equilibrium behavior of the terrorist is not always decreasing (or increasing) with security. This is driven by the fact that direct and indirect results of increasing security work in opposite directions. The direct result is what we mentioned above: terrorist effort decreases assuming that the size of population attending the public place is constant. The indirect result means that the mass of population attending the public place increases (Proposition A1) which motivates the terrorist to increase effort as the public place becomes a more valuable target in the eyes of terrorist (i.e., $\frac{\partial \hat{t}}{\partial m} > 0$, see Lemma 2 in the Appendix). Consequently, whether terrorist effort increases or decreases with government security level depends on which effect is greater.

We present two numerical examples whose only purpose is to indicate that the inequality in Proposition A2 might and might not hold.

Example 3.1.

Case 1 in Proposition A2: $\frac{d\hat{t}(s)}{ds} \geq 0$ for each s . Assume that $\lambda(s, t) = \frac{1}{2(1+s)}(1 - e^{-t}) + \frac{1}{2}$ and α is uniformly distributed. Note that $\lambda(s, t) \in [0.5, 1]$ for each pair (s, t) . In order to determine the sign of $\frac{d\hat{t}(s)}{ds}$, we compute $F(1 - \lambda) \cdot \lambda_{ts} - F_\alpha(1 - \lambda) \cdot \lambda_s \cdot \lambda_t = \frac{1}{2}(1 + s)^{-2}e^{-t}(2\lambda - 1)$. Since $\lambda \geq \frac{1}{2}$, it is true that $F(1 - \lambda) \cdot \lambda_{ts} - F_\alpha(1 - \lambda) \cdot \lambda_s \cdot \lambda_t \geq 0$. Consequently, $\frac{d\hat{t}(s)}{ds} \geq 0$ for each s .

Example 3.2.

Case 2 in Proposition A2: $\frac{d\hat{t}(s)}{ds} < 0$ for each s . Assume that $\lambda(s, t) = \frac{1}{2(1+s)}(1 - e^{-t})$ and, again,

α is uniformly distributed. Note that $\lambda(s, t) \in [0, 0.5]$ for each pair (s, t) . In order to determine the sign of $\frac{d\tilde{t}(s)}{ds}$, we compute $F(1 - \lambda) \cdot \lambda_{ts} - F_\alpha(1 - \lambda) \cdot \lambda_s \cdot \lambda_t = \frac{1}{2}(1 + s)^{-2}e^{-t}(2\lambda - 1)$. Since $\lambda \leq \frac{1}{2}$, it is true that $F(1 - \lambda) \cdot \lambda_{ts} - F_\alpha(1 - \lambda) \cdot \lambda_s \cdot \lambda_t \leq 0$. Consequently, $\frac{d\tilde{t}(s)}{ds} \leq 0$ for each s .

Proposition A3 is important because it shows that even if terrorist effort increases in response to increased government security (as indicated as a possibility in Proposition A2) the overall impact is that the probability of a successful attack goes strictly down. This is a desired result.

4 Effectiveness of Counter-Terrorism Security

We arrive at the core analysis of our paper: the effectiveness of counter-terrorism security. Our main focus is the impact of counter-terrorism security on the expected number of casualties $\tilde{m}(s) \cdot \tilde{\lambda}(s)$. Intuitively, it might seem that more security always leads to a decrease in the number of casualties. In Proposition B, which is the main result of our paper, we show that this is not the true.

In order to establish the second part of Proposition B, we assume the log-concavity of F .¹²

Assumption 5.

F is log-concave.

Proposition B.

1. $\frac{d\tilde{m}(s) \cdot \tilde{\lambda}(s)}{ds} \leq (\geq) 0$ for each s , if $F(1 - \lambda) - F_\alpha(1 - \lambda) \cdot \lambda \leq (\geq) 0$ for each pair (s, t) .
2. If we add Assumption 5, then one and only one of the following is true

Case 1. $\frac{d\tilde{m}(s) \cdot \tilde{\lambda}(s)}{ds} \leq 0$ for each s .

Case 2. $\frac{d\tilde{m}(s) \cdot \tilde{\lambda}(s)}{ds} \geq 0$ for each s .

Case 3. There exists security level \bar{s} such that $\frac{d\tilde{m}(s) \cdot \tilde{\lambda}(s)}{ds} \geq 0$ for each $s < \bar{s}$ and $\frac{d\tilde{m}(s) \cdot \tilde{\lambda}(s)}{ds} \leq 0$ for each $s > \bar{s}$.

¹²Log-concavity is a standard and commonly made assumption in the literature (see Bagnoli and Bergstrom (2005)) and implies that $\frac{F_\alpha}{F}$ is a decreasing function. Several common distribution functions (e.g., normal, uniform, exponential) are log-concave.

The intuition for Proposition B1 comes from the fact that increased security causes (1) a reduction in the equilibrium probability of a successful attack (Proposition A3) and (2) an increase in the equilibrium mass of population attending the public place (Proposition A1). These two effects work in opposite directions and it is not clear which effect is stronger. Hence, it might be the case that increases security encourages too many individuals to attend the public place which negates the effect of the lower probability of a successful attack and leads to a higher expected damage from terrorism.

While Proposition B1 already shows that it is not true that more is always better, this result lacks some regularity. After all, Proposition B1 does not reject the possibility that the expected number of casualties initially increases, then decreases, then increases again, and so on. In other words, it is possible that the range of values of security s looks like a patchwork with each segment having different behavior of the expected number of casualties compared to the adjacent segments.

Our goal is to achieve some form of regularity. Ideally, we want the range of values of s to be divided into at most two segments, one on which the expected number of casualties is increasing and one on which the expected number of casualties is decreasing. It turns out that adding a very general Assumption 5 does the job.

Proposition B2 shows that there are just three cases describing the relationship between counter-terrorism security and expected number of casualties: expected number of casualties is always decreasing in security, expected number of casualties is always increasing in security and lastly, expected number of casualties is first increasing and then decreasing in security. Note that Case 1 and Case 2 are not implied by Case 3 since there maybe cases where \bar{s} does not exist.

Next, we provide a numerical example for Proposition B2.

Example 4.1. 1. $\lambda(s, t) = \frac{1}{1+s}t + x$ where

- For Case 1, we assume $x = 0$
- For Case 2, we assume $x = 0.75$
- For Case 3, we assume $x = 0.25$

2. Cost function $c(t) = \frac{1}{2}t^2$.

3. α is uniformly distributed.

Note that $\tilde{t}(s) = \frac{(1-x)(1+s)}{(1+s)^2+1}$, $\tilde{m}(s) = \frac{(1-x)(1+s)^2}{(1+s)^2+1}$, and $\tilde{\lambda}(s) = \frac{x(1+s)^2+1}{(1+s)^2+1}$. It is easy to check that $\tilde{m}(s) \cdot \tilde{\lambda}(s)$ is strictly decreasing if $x = 0$ and strictly increasing if $x = 0.75$. If $x = 0.25$, then $\tilde{m}(s) \cdot \tilde{\lambda}(s)$ is strictly increasing up to $\bar{s} = 0.4142$ and strictly decreasing for $s > \bar{s}$.

The first case captures the expected and desired impact of security: more is always better. The second case is the extreme opposite of Case 1. It is an unexpected result as Case 2 indicates that even under costless security the optimal security level should be zero because more security will always result with more casualties. In Case 2, more is always worse.

Case 3 is between the two extremes depicted in Case 1 and Case 3. We find this case especially interesting and important. For small values of government security, the expected impact of terror is rising, rather than decreasing, in government security. However, this expectation is decreasing for large enough values of government security. That is, the counter-productive security can happen only when that security is too small.

Our last result analyzes the impact of security on total welfare of the population. Total welfare sums over the welfare for all members of the population and the next proposition shows that increasing government security increases the welfare.

Proposition C.

Total welfare for the population weakly increases in security s for each s .

Proposition C can be intuitively explained in the following way. When security increases, then our population can be divided into three groups. First, there are people who were attending the public place before the increase of security. They continue attending the public place and their total welfare increases as the probability of successful attack decreases (Proposition A3). Second, due to Proposition A1, we know that there is a group of people who switch from staying away to attending the public place. For this group, the total welfare also increases; after all, the opt for attending the public place because this is a more attractive alternative compared to staying

away. Finally, the third group: people who continue staying away after security increased. For them, nothing changes in terms of utility. Overall, we note that increasing security either increases utility or keeps utility at the same level. Consequently, total welfare is not strictly decreasing.

4.1 Policy Implications

Results in Proposition B2, especially Case 3, are very important from the policy perspective. Contrary to what intuition might suggest, Proposition B2 tells that the expected number of casualties need not always be decreasing in security; an increase is also plausible.

If Case 3 is the one that captures reality, then knowing the threshold security level, \bar{s} , implies the certainty that increasing security beyond that threshold value will always, as desired, decrease the expected number of casualties. In short, Case 3 says that it is better to over-spend on counter-terrorism than under-spend.

Overspending has been found in the game-theoretic literature looking at defensive security in Lapan and Sandler (1988) and Sandler and Siqueira (2006). We show that overspending might in fact be optimal even if there is no other country to which the terror attack might be diverted. Our results also highlight that if the government is constrained as to the amount of security it can provide, such that its funds allow provision of $s < \bar{s}$ then it might be optimal to provide no security at all. Some security can in fact be more harmful than no security.

We feel that the results in our paper can help rationalize the varying security levels chosen by countries facing a similar terror risk. We show that a government with heavy resource constraints should always choose a very low level of security whereas a government with no significant constraints should choose a security level high enough to negate any incentives to increase the terror effort steaming from the greater mass of population attending the public place.

5 Conclusions

Terrorism is one of the biggest concerns of the modern world and has a significant impact on our daily behaviors. In response to the threat of terrorism, governments spend resources on providing counter-terrorism security. In this paper, we analyze the effectiveness of this security as measured by the expected number of casualties due to terror attack.

We make a theoretical contribution to the literature by the addition of a strategic population. We find that increasing security need not lead to a decrease of the expected number of casualties. In fact, it is likely that this number would decrease; that is, more security causes more harm. It also is possible that this unexpected outcome happens only when security is too small.

Our results have important policy implications. In the case of the government which wants to reduce the expected impact of terror but operates under constraints (i.e., the limit of how much security can be provided is low), we show that less security might in fact be better if the government is unable to provide infinite security. However, even if there are no severe constraints, the government must be cautious not to be too prudent as over-spending on counter-terrorism security could be significantly better than under-spending.

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A Appendix: Proofs

Preliminary Results. We start with results that will help us prove the main propositions.

Recall that, for a fixed s , $\hat{m}(s, t)$ denotes population's best-response function to terror effort t . Equation (3) depicts \hat{m} and Lemma 1 describes the behavior of \hat{m} .

Lemma 1.

a) For each s , $\frac{\partial \hat{m}}{\partial t} \leq 0$.

b) For each t , $\frac{\partial \hat{m}}{\partial s} \geq 0$.

Proof of Lemma 1.

(a) Observe the following.

$$\frac{\partial \hat{m}}{\partial t} = -F_\alpha(1 - \lambda) \cdot \lambda_t \tag{7}$$

Since $F_\alpha \geq 0$ and $\lambda_t > 0$ (Assumption 1), we conclude that $\frac{\partial \hat{m}}{\partial t} \leq 0$.

(b) Observe the following.

$$\frac{\partial \hat{m}}{\partial s} = -F_\alpha(1 - \lambda) \cdot \lambda_s \tag{8}$$

Since $F_\alpha \geq 0$ and $\lambda_s < 0$ (Assumption 1), we conclude that $\frac{\partial \hat{m}}{\partial s} \geq 0$. ■

Recall that, for a fixed s , $\hat{t}(s, m)$ denotes terrorist's best-response function to population's mass m . Equation (4) depicts \hat{t} and Lemma 2 describes the behavior of \hat{t} .

Lemma 2.

a) For each s , $\frac{\partial \hat{t}}{\partial m} > 0$.

b) For each m , $\frac{\partial \hat{t}}{\partial s} < 0$.

Proof of Lemma 2.

(a) Observe the following.

$$\frac{\partial \hat{t}}{\partial m} = \frac{-\lambda_t}{m \cdot \lambda_{tt} - c''} \quad (9)$$

Since $\lambda_t > 0$, $\lambda_{tt} \leq 0$, and $c'' > 0$ (Assumption 1), we conclude that $\frac{\partial \hat{t}(s,m)}{\partial m} > 0$.

(b) Observe the following.

$$\frac{\partial \hat{t}}{\partial s} = \frac{-m \cdot \lambda_{ts}}{m \cdot \lambda_{tt} - c''} \quad (10)$$

Since $\lambda_{ts} < 0$, $\lambda_{tt} \leq 0$, and $c'' > 0$ (Assumption 1), we conclude that $\frac{\partial \hat{t}(s,t)}{\partial s} < 0$. ■

Recall that the system of two equations (5) and (6) depicts the second-stage equilibrium values $\tilde{m}(s)$ and $\tilde{t}(s)$ which we analyze in Proposition A. In order to prove these propositions, we need the first-order derivatives of \tilde{m} and \tilde{t} with respect to s .

$$\frac{d\tilde{m}}{ds} = \frac{\partial \hat{m}}{\partial s} + \frac{\partial \hat{m}}{\partial t} \cdot \frac{d\tilde{t}}{ds} \quad (11)$$

$$\frac{d\tilde{t}}{ds} = \frac{\partial \hat{t}}{\partial s} + \frac{\partial \hat{t}}{\partial m} \cdot \frac{d\tilde{m}}{ds} \quad (12)$$

We re-write the above in the following way.

$$\left[1 - \frac{\partial \hat{m}}{\partial t} \frac{\partial \hat{t}}{\partial m} \right] \frac{d\tilde{m}}{ds} = \frac{\partial \hat{m}}{\partial s} + \frac{\partial \hat{m}}{\partial t} \cdot \frac{\partial \hat{t}}{\partial s} \quad (13)$$

$$\left[1 - \frac{\partial \hat{t}}{\partial m} \frac{\partial \hat{m}}{\partial t} \right] \frac{d\tilde{t}}{ds} = \frac{\partial \hat{t}}{\partial s} + \frac{\partial \hat{t}}{\partial m} \cdot \frac{\partial \hat{m}}{\partial s} \quad (14)$$

To help with the proofs, we re-write $\frac{\partial \hat{t}}{\partial m}$ and $\frac{\partial \hat{t}}{\partial s}$ in the following way.

$$\frac{\partial \hat{t}}{\partial m} = \frac{\lambda_t^2}{c'' \cdot \lambda_t - c' \cdot \lambda_{tt}} \quad (15)$$

$$\frac{\partial \hat{t}}{\partial s} = \frac{c' \cdot \lambda_{ts}}{c'' \cdot \lambda_t - c' \cdot \lambda_{tt}} \quad (16)$$

In order to simplify the notation, we write F and F_λ instead of $F(1-\lambda)$ and $F_\alpha(1-\lambda)$, respectively.

Proof of Proposition A. We prove (1). To prove that $\frac{d\tilde{m}(s)}{ds} \geq 0$ for each s , we analyze $\frac{d\tilde{m}}{ds}$ by looking at equation (13).

(i) Left-hand side of (13). Since $\frac{\partial \tilde{m}}{\partial t} \leq 0$ (Lemma 1) and $\frac{\partial \tilde{t}}{\partial m} > 0$ (Lemma 2), we conclude that $\frac{\partial \tilde{m}}{\partial t} \frac{\partial \tilde{t}}{\partial m} \leq 0$. Hence, $\left[1 - \frac{\partial \tilde{m}}{\partial t} \frac{\partial \tilde{t}}{\partial m}\right] > 0$. This implies that the sign of $\frac{d\tilde{m}}{ds}$ is the same as sign of the right-hand side of equation (13).

(ii) Right-hand side of (13). Since $\frac{\partial \tilde{m}}{\partial s} \geq 0$ (Lemma 1), $\frac{\partial \tilde{m}}{\partial t} \leq 0$ (Lemma 1), and $\frac{\partial \tilde{t}}{\partial s} < 0$ (Lemma 2), we conclude that the right-hand side of equation (13) is at least zero. From (i) and (ii), we conclude that $\frac{d\tilde{m}}{ds} \geq 0$.

We prove (2). We only show that $\frac{d\tilde{t}(s)}{ds} \geq 0$ for each s if $F \cdot \lambda_{ts} - F_\alpha \cdot \lambda_s \cdot \lambda_t \geq 0$ for each pair (s, t) as the proof for inverse inequalities follows the same reasoning. We derive $\frac{d\tilde{t}}{ds}$ from equation (14).

(i) Left-hand side of (14). Since $\frac{\partial \tilde{t}}{\partial m} > 0$ (Lemma 2) and $\frac{\partial \tilde{m}}{\partial t} \leq 0$ (Lemma 1), we conclude that $\frac{\partial \tilde{t}}{\partial m} \frac{\partial \tilde{m}}{\partial t} \leq 0$. Hence, $\left[1 - \frac{\partial \tilde{t}}{\partial m} \frac{\partial \tilde{m}}{\partial t}\right] > 0$. This implies that the sign of $\frac{d\tilde{t}}{ds}$ is the same as the sign of the right-hand side of equation (14).

(ii) Right-hand side of (14). In order to determine the sign of the right-hand side of equation (14), we use equations (8), (15), and (16) and write $\frac{\partial \tilde{t}}{\partial s} + \frac{\partial \tilde{t}}{\partial m} \frac{\partial \tilde{m}}{\partial s} = \frac{c' \cdot \lambda_{ts} - F_\alpha \cdot \lambda_s \cdot \lambda_t^2}{c'' \cdot \lambda_t - c' \cdot \lambda_{tt}}$. Given Assumptions 1 and 2, we conclude that the sign of $\frac{d\tilde{t}}{ds}$ is the same as the sign of $c' \cdot \lambda_{ts} - F_\alpha \cdot \lambda_s \cdot \lambda_t^2$. From the terrorist's first-order condition, we know that $c' = \tilde{m} \cdot \lambda_t = F \cdot \lambda_t$. Hence, $c' \cdot \lambda_{ts} - F_\alpha \cdot \lambda_s \cdot \lambda_t^2 = \lambda_t \cdot [F \cdot \lambda_{ts} - F_\alpha \cdot \lambda_s \cdot \lambda_t]$. Since $\lambda_t > 0$ (Assumption 1), we conclude that if $F \cdot \lambda_{ts} - F_\alpha \cdot \lambda_s \cdot \lambda_t \geq 0$ for each pair (s, t) , then $\frac{d\tilde{t}}{ds} \geq 0$ for each s .

We prove (3). Observe that $\frac{d\tilde{\lambda}}{ds} = \lambda_s + \lambda_t \frac{d\tilde{t}}{ds}$. We use equation (12), and write $\frac{d\tilde{\lambda}}{ds} = \lambda_s + \lambda_t \left[\frac{\partial \tilde{t}}{\partial s} + \frac{\partial \tilde{t}}{\partial m} \cdot \frac{d\tilde{m}}{ds} \right]$. We use equations (7), (8), and (11) and write $\frac{d\tilde{m}}{ds} = -F_\alpha \cdot \left[\lambda_s + \lambda_t \frac{d\tilde{t}}{ds} \right] = -F_\alpha \cdot \frac{d\tilde{\lambda}}{ds}$. Hence, $\left[1 + \lambda_t \cdot \frac{\partial \tilde{t}}{\partial m} \cdot F_\alpha\right] \frac{d\tilde{\lambda}}{ds} = \lambda_s + \lambda_t \cdot \frac{\partial \tilde{t}}{\partial s}$. Because of Assumption 1 and Lemma 2, we know that $\left[1 + \lambda_t \cdot \frac{\partial \tilde{t}}{\partial m} \cdot F_\alpha\right] > 0$ and $\lambda_s + \lambda_t \cdot \frac{\partial \tilde{t}}{\partial s} < 0$. We conclude that $\frac{d\tilde{\lambda}}{ds} < 0$ for each s . ■

Proof of Proposition B. We prove (1). We only show that $\frac{d\tilde{m}(s) \cdot \tilde{\lambda}(s)}{ds} \geq 0$ for each s if $F - F_\alpha \cdot \lambda \leq 0$ for each pair (s, t) as the proof of inverse inequalities follows the same reasoning. Note that

$\frac{d\tilde{m}\tilde{\lambda}}{ds} = \frac{d\tilde{m}}{ds}\tilde{\lambda} + \tilde{m}\frac{d\tilde{\lambda}}{ds}$. From the proof of Proposition A, we know that $\frac{d\tilde{m}}{ds} = -F_\alpha \cdot \frac{d\tilde{\lambda}}{ds}$. Hence, we write $\frac{d\tilde{m}\tilde{\lambda}}{ds} = \frac{d\tilde{\lambda}}{ds} \left[F - F_\alpha \cdot \tilde{\lambda} \right]$. Since $\frac{d\tilde{\lambda}}{ds} < 0$ (Proposition A), we know that the sign of $\frac{d\tilde{m}\tilde{\lambda}}{ds}$ depends on the sign of $\left[F - F_\alpha \cdot \tilde{\lambda} \right]$. We conclude that if $F - F_\alpha \cdot \lambda \leq 0$ for each pair (s, t) , then $\frac{d\tilde{m}\tilde{\lambda}}{ds} \geq 0$ for each s .

We prove (2). From (1), we know that the expected number of casualties is weakly increasing if and only if $\frac{1}{\lambda} \leq \frac{F_\alpha(1-\tilde{\lambda})}{F(1-\tilde{\lambda})}$. From Proposition A, we know that $\tilde{\lambda}$ is a strictly decreasing function. This implies that the left-hand side of this inequality is a strictly increasing function. As of the right-hand side of this inequality, note that, since $1 - \tilde{\lambda}$ is an increasing function, it is true that $F(1 - \tilde{\lambda})$ is also an increasing function in s . Because of Assumption 5, we know that $\frac{F_\alpha(1-\tilde{\lambda})}{F(1-\tilde{\lambda})}$ is a decreasing function in s .

Hence, in the inequality $\frac{1}{\lambda} \leq \frac{F_\alpha(1-\tilde{\lambda})}{F(1-\tilde{\lambda})}$ we have a strictly increasing function on the left-hand side and, on the right-hand side, a decreasing function. These functions can cross at most (if at all) only once. If they do not cross, then the inequality in question either holds for each s (i.e., the expected number of casualties always increases) or does not hold for any s (i.e., the expected number of casualties always decreases). However, if these functions cross at some \bar{s} , then $\frac{1}{\lambda} \leq \frac{F_\alpha(1-\tilde{\lambda})}{F(1-\tilde{\lambda})}$ for all $s < \bar{s}$ and $\frac{1}{\lambda} \geq \frac{F_\alpha(1-\tilde{\lambda})}{F(1-\tilde{\lambda})}$ for all $s > \bar{s}$. That is, the expected number of casualties initially increases to start decreasing for $s > \bar{s}$. ■

Proof of Proposition C. Let $\tilde{U}_1(s) := U_1(s, \tilde{t}(s), \tilde{m}(s))$, $\tilde{U}_2(s) := U_2(s, \tilde{t}(s), \tilde{m}(s))$, and $\tilde{U}_{TW}(s) := \tilde{U}_1(s) + \tilde{U}_2(s)$. We want to prove $\frac{d\tilde{U}_{TW}}{ds} \geq 0$ for each s . First, we compute $\frac{d\tilde{U}_1}{ds} = \frac{d\tilde{m}}{ds} \cdot \left[1 - \tilde{\lambda} \right] - \tilde{m} \cdot \frac{d\tilde{\lambda}}{ds}$. From the proof of Proposition A, we know that $\frac{d\tilde{m}}{ds} = -F_\alpha \cdot \frac{d\tilde{\lambda}}{ds}$. Hence, we write $\frac{d\tilde{U}_1}{ds} = -F_\alpha \cdot \frac{d\tilde{\lambda}}{ds} \cdot \left[1 - \tilde{\lambda} \right] - F \cdot \frac{d\tilde{\lambda}}{ds}$. Second, using the Leibniz rule, we compute $\frac{d\tilde{U}_2}{ds} = F_\alpha \cdot \frac{d\tilde{\lambda}}{ds} \cdot \left[1 - \tilde{\lambda} \right]$. Consequently, $\frac{d\tilde{U}_{TW}}{ds} = -F \cdot \frac{d\tilde{\lambda}}{ds}$. Since $\frac{d\tilde{\lambda}}{ds} < 0$ (Proposition A), we conclude that $\frac{d\tilde{U}_{TW}}{ds} \geq 0$ for each s . ■